COURSE HANDOUT

Course Code	ACSC13
Course Name	Design and Analysis of Algorithms
Class / Semester	IV SEM
Section	A-SECTION
Name of the Department	CSE-CYBER SECURITY
Employee ID	IARE11023
Employee Name	Dr K RAJENDRA PRASAD
Topic Covered	Job sequencing with deadlines
Course Outcome/s	Compare Identify suitable problem solving techniques for a given problem and finding optimized solutions using Greedy Method
Handout Number	27
Date	

Content about topic covered: Job Sequencing with Deadlines

Job Sequencing with Deadlines

We are given a set of n jobs. Associated with job i is an integer deadline $d_i \geq 0$ and a profit $P_i > 0$. For any job i the profit P_i is earned if and only if the job is completed by its deadline. To complete the job, one has to process the job on a machine for one unit of time. Only one machine is available for processing jobs.

A feasible solution for this problem is a subset J of jobs such that each job in this subset can be completed by its deadline. The value of a feasible solution J is the sum of the profits of the jobs in J, or

$$\sum_{i \in J} P_i$$

An optimal solution is feasible solution with maximum value.

E.g.: Let n = 4, $(P_1,P_2,P_3,P_4) = (100,10,15,27)$ and $(d_1,d_2,d_3,d_4)= (2,1,2,1)$
The feasible solutions and their values are:

S.No.	Feasible solution	Processing sequence	value
1	(1,2)	2,1	110
2	(1,3)	1,3 or 3,1	115
3	(1,4)	4,1	127
4	(2,3)	2,3	25
5	(3,4)	4,3	42
6	(1)	1	100
7	(2)	2	10
8	(3)	3	15
9	(4)	4	27

Solution 3 is optimal. In this solution, only Jobs 1 and 4 are processed and the value is 127. These jobs must be processed the order job 4 followed by job 1.

Here the objective function is $\sum_{i\in J} P_i$

To include the next job that increases $\sum_{i\in J}(P_i)$ subject to the constraint that J is a feasible solution.
Consider the jobs in descending order of P_i.

J = ∅	$\sum_{i\in J} P_i$
J= {1}	feasible
J= {1,4}	feasible
J= {1,3,4}	not feasible
J= {1,2,4}	not feasible

So J= {1,4}

Value = 127.

Eg: Let n = 5, $(P_1,P_2,P_3,P_4,P_5) = (20,15,10,5,1)$ and $(d_1,d_2,d_3,d_4,d_5)= (2,2,1,3,3)$

J	Assigned slots	Job considered	Action	Profit
∅	None	1	Assign to [1,2]	0
{1}	[1,2]	2	Assign to [0,1]	20
{1,2}	[0,1],[1,2]	3	Cannot fit. Reject	35
{1,2}	[0,1],[1,2]	4	Assign to [2,3]	35
{1,2,4}	[0,1],[1,2],[2,3]	5	reject	40

Optimal solution is J={1,2,4} with a profit of 40.

Greedy algorithm for sequencing unit time jobs with deadlines and profits:

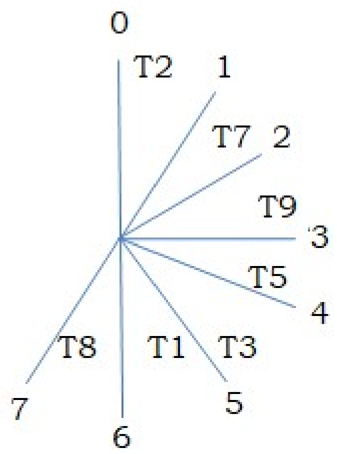
```
Algorithm JS(d, j, n)
// d[i] ≥ 1, 1 ≤ i ≤ n are the deadlines, n ≥ 1. The jobs
// are ordered such that p[1] ≥ p[2] ≥ ··· ≥ p[n]. J[i]
// is the ith job in the optimal solution, 1 ≤ i ≤ k.
// Also, at termination d[J[i]] ≤ d[J[i + 1]], 1 ≤ i < k.
{
    d[0] := J[0] := 0; // Initialize.
    J[1] := 1; // Include job 1.
    k := 1;
    for i := 2 to n do
    {
        // Consider jobs in nonincreasing order of p[i]. Find
        // position for i and check feasibility of insertion.
        r := k;
        while ((d[J[r]] > d[i]) and (d[J[r]] ≠ r)) do r := r − 1;
        if ((d[J[r]] ≤ d[i]) and (d[i] > r)) then
        {
            // Insert i into J[ ].
            for q := k to (r + 1) step −1 do J[q + 1] := J[q];
            J[r + 1] := i; k := k + 1;
        }
    }
    return k;
}
```

Example:

job	d	P
T1	7	15
T2	2	20
T3	5	30
T4	3	18
T5	4	18
T6	5	10
T7	2	23
T8	7	16
T9	3	25

After arranging them in decreasing order of the profit

Job	d	P
T3	5	30
T9	3	25
T7	2	23
T2	2	20
T4, T5	3, 4	18,18
T8	7	16
T1	7	15
T6	5	10



Total Profit = 20+23+25+18+30+15+16 = 147.